AP Calculus BC — WS48-Unit 6 Review

1) $v(t) = 7t^6 - 4t^2 + 12$

$s(t) = t^7 - \frac{4}{3}t^3 + 12t + C$

$24 = 1 - \frac{4}{3} + 12 + C$

$C = \frac{37}{3}$

$s(t) = t^7 - \frac{4}{3}t^3 + 12t + \frac{37}{3}$

2) $\int \frac{4x^4+3}{4x^5+15x+2}\,dx$

$\int (4x^4+3)(4x^5+15x+2)^{-1}\,dx$

$\boxed{\frac{1}{5}\ln(4x^5+15x+2) + C}$

3) $\int_0^1 (x^3+x)(x^4+2x^2+9)^{1/2}\,dx$

$u = x^4+2x^2+9$ $\quad u(0) = 9$

$du = (4x^3+4x)\,dx$ $\quad u(1) = 12$

$\frac{du}{(4x^3+4x)} = dx$

$\frac{1}{4}\int_9^{12} u^{1/2}$

$\frac{1}{6}u^{3/2}\Big]_9^{12}$

$\boxed{\frac{1}{6}(12)^{3/2} - \frac{9}{2}}$

4) $\int x^2 \sin x\,dx$

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$+x^2$	$\sin x$
$-2x$	$-\cos x$
$+2$	$-\sin x$
0	$\cos x$

$-x^2\cos x + 2x\sin x + 2\cos x + C$

5) $\int_1^2 (9x^2-4x+1)\ln x\,dx$

$u = \ln x$ $\quad v = 3x^3-2x^2+x$

$du = \frac{1}{x}dx$ $\quad dv = (9x^2-4x+1)\,dx$

$\ln x(3x^3-2x^2+x)\Big|_1^2 - \int_1^2 (3x^2-2x+1)\,dx$

$[\ln 2\,(18)] - 0 \quad - [x^3-x^2+x]_1^2$

$18\ln 2 \quad - (5-1) = \boxed{18\ln 2 - 4}$

6) $\int \frac{4}{x^2+3x+2}\,dx = \int \frac{4}{(x+2)(x+1)}\,dx$

$\frac{A}{x+2} + \frac{B}{x+1} = \frac{4}{(x+2)(x+1)}$

$A(x+1) + B(x+2) = 4$

$x=-1$: $B = 4$

$x=-2$: $-A = 4$, $A = -4$

$\int \left[\frac{4}{x+1} - \frac{4}{x+2}\right] dx$

$4\ln|x+1| - 4\ln|x+2| + C$

7) $\int_0^1 \frac{1}{1-x}\,dx$

$\lim_{b\to 1^-} \int_0^b \frac{1}{1-x}\,dx$

$\lim_{b\to 1^-} \left[-\ln|1-x|\right]_0^b$

$\lim_{b\to 1^-} \left[-\ln|1-b| + \ln 1\right]$

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8) $\int_1^\infty \frac{x}{(1+x^2)^2}\,dx = \lim_{b\to\infty}\int_1^b x(1+x^2)^{-2}\,dx$

$\lim_{b\to\infty}\left[-\frac{1}{2}(1+x^2)^{-1}\right]_1^b$

$\lim_{b\to\infty}\left[-\frac{1}{2(1+b^2)} + \frac{1}{4}\right] = \boxed{\frac{1}{4}}$

9) $$\int_{-1}^{1} \frac{3x^2+2x+1}{x+4}\,dx$$

$$\begin{array}{r} 3x - 10 + \frac{41}{x+4} \\ x+4\,\overline{)\,3x^2+2x+1} \\ -(3x^2+12x) \\ \hline -10x+1 \\ -10x-40 \\ \hline 41 \end{array}$$

$$\int_{-1}^{1}\left[3x-10+\frac{41}{x+4}\right]dx$$

$$\left[\frac{3}{2}x^2-10x+41\ln(x+4)\right]_{-1}^{1}$$

$$\left[\frac{3}{2}-10+41\ln 5\right]-\left[\frac{3}{2}+10+41\ln 3\right]$$

10) $$\int_{-2}^{8}(3g(x)+2)=35$$

$$\int_{5}^{-2} g(x)dx=-12$$

$$3\left[\int_{-2}^{5} g(x)dx+\int_{5}^{8} g(x)dx\right]+\int_{-2}^{8}2dx=35$$

$$3\left[12+\int_{5}^{8} g(x)dx\right]+20=35$$

$$36+3\int_{5}^{8} g(x)dx=15$$

$$3\int_{5}^{8} g(x)dx=-21$$

$$\boxed{\int_{5}^{8} g(x)dx=-7}$$

11) $$\lim_{h\to 0} f(x)=-\sin x$$

12) $$\lim_{x\to\infty}\frac{\ln(3x+5)}{\ln(2x^2-1)}$$

$$\lim_{x\to\infty}\ln(3x+5)=\infty$$

$$\lim_{x\to\infty}\ln(2x^2-1)=\infty$$

Using L'Hopital's Rule:

$$\lim_{x\to\infty}\frac{\frac{3}{3x+5}}{\frac{4x}{2x^2-1}}$$

$$\lim_{x\to\infty}\left[\frac{3}{3x+5}\cdot\frac{2x^2-1}{4x}\right]$$

$$\lim_{x\to\infty}\left[\frac{6x^2-3}{12x^2+20x}\right]=\frac{1}{2}$$